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Influence of lateral conduction due to flow temperature variations in transient heat transfer measurements

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Abstract

An analytical model is presented addressing the effect of lateral conduction due to varying adiabatic wall temperature distributions in transient film cooling experiments using thermochromic liquid crystals (TLC). Applying the analysis for a typical experimental situation shows, that results evaluated without taking the lateral conduction effect into account can lead to erroneous results especially in the regions of high film cooling effectiveness. An alternative data evaluation procedure is suggested considering the lateral conduction effects based on the given analysis.

1. Introduction

Transient heat transfer measurement techniques using thermochromic liquid crystals (TLC) have been extensively used in obtaining detailed heat transfer and/or adiabatic wall temperature distributions for many internal and external flow applications (see e.g., the reviews by Ireland and Jones [1] and Ekkad and Han [2]). Usually a one-dimensional conduction/convection model is applied to analyze the measurement data assuming a semi-infinite solid and determining the unknown parameters. In case of a transient heat transfer experiment for which the driving fluid (adiabatic wall) temperature T_{aw}, the initial temperature T_0 and the thermal properties (a,k) of the wall material are known, the solution

$$\frac{T_w - T_0}{T_{aw} - T_0} = 1 - \exp\left(\frac{h^2at}{k^2}\right)\operatorname{erfc}\left(\frac{h\sqrt{at}}{k}\right) \tag{1}$$

is used to determine the unknown heat transfer coefficient h from the measurement of the wall temperature T_w at a certain time t as indicated e.g., by narrow-band TLC. Thereby it is assumed, that the conduction process within the wall is one-dimensional and that lateral conduction effects can be neglected. The effect of lateral conduction on the results for spatially varying heat transfer distributions including anisotropic material properties was first analyzed by Vedula et al. [3] using two-dimensional numerical finite-element computations. An approximate analysis for this case was recently given by Kingsley-Rowe et al. [4], deriving a correction parameter, which can be used in a post-processing manner on the measurement data starting with Eq. (1) as the initial approximation. A series of finite-difference computations were used to derive a correction correlation together with a correction procedure depending on typical experimental parameters. Thereby periodic variations of the heat transfer coefficients were assumed. More general approaches couple directly the data evaluation to three-dimensional numerical models determining the full surface heat transfer distribution by applying 3D inverse procedures. Using a three-dimensional unsteady heat conduction solver an iterative scheme for the determination of the heat transfer coefficients has been proposed by Lin and Wang [5] and applied to impingement jet arrays for confined jets at different jet Reynolds numbers by Wang et al. [6].

In film cooling investigations the adiabatic wall temperature is not known a priori. Therefore two unknowns arise

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Nomenclature

a	wall thermal diffusivity	η	dimensionless lateral coordinate
A_n, C_n, c_n	parameters	η_{aw}	film cooling effectiveness
Bi	Biot number	ξ	dimensionless vertical coordinate
h	heat transfer coefficient	τ	dimensionless time – Fourier number
k	wall thermal conductivity	θ	temperature difference
L	length		
p	Laplace variable	*Subscripts*	
t	time	aw	adiabatic wall
T	temperature	C	coolant
x	vertical coordinate	G	hot gas
y	lateral coordinate	W	wall
ε_W	wall cooling effectiveness	0	initial

at each position on the surface, namely the heat transfer coefficient h and the adiabatic wall temperature T_{aw} (or film cooling effectiveness η_{aw}). The one-dimensional approach to this situation was first described by Vedula and Metzger [7] using two different TLCs in a single experiment or two separate transient tests, leading to the equations

$$\frac{T_{w1}-T_0}{T_{aw}-T_0} = 1-\exp\left(\frac{h^2 a t_1}{k^2}\right)\operatorname{erfc}\left(\frac{h\sqrt{a t_1}}{k}\right)$$
$$\frac{T_{w2}-T_0}{T_{aw}-T_0} = 1-\exp\left(\frac{h^2 a t_2}{k^2}\right)\operatorname{erfc}\left(\frac{h\sqrt{a t_2}}{k}\right) \quad (2)$$

to evaluate at each position the two unknowns h and T_{aw} from two wall temperature indications.

In this case lateral conduction will occur due to both, lateral variations in the heat transfer coefficient and in adiabatic wall temperature. Because of the simultaneous evaluation, the lateral conduction effects will influence both parameters directly. A data processing technique for such film cooling situations has been proposed by Ling et al. [8] using a three-dimensional unsteady finite-difference approach. Having the need for an iteration of both parameters at each position, the computational effort can become quite large. Ling et al. [8] showed, that the effect of lateral conduction due to adiabatic wall temperature variations can be more significant than due to variations in the heat transfer distribution for such a situation and will influence the evaluated data for both parameters. The aim of the current work is to provide an analytical background for this effect using the simplified assumptions of constant heat transfer coefficient and two-dimensional behavior. The given model can also be extended to two-dimensional surface variations in adiabatic wall temperature to account additionally for longitudinal conduction, which is important especially around the film cooling holes and possibly combined with simplified approaches as given by Kingsley-Rowe et al. [4] for spatially varying heat transfer distributions.

2. Analytical model

For the present investigation the following model will be used. A semi-infinite strip ($0 \leqslant x < \infty; 0 \leqslant y \leqslant L$) has a homogenous initial temperature T_0, constant material properties and will be subjected to a convective boundary condition at $x = 0$ for $t > 0$ with a constant heat transfer coefficient h and a spatially varying fluid temperature $T_{aw}(y)$ as shown in Fig. 1.

The heat conduction problem for the solid can be described by:

$$\frac{1}{a}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \quad (3)$$

with the initial condition: $T(x, y, t = 0) = T_0$ (3a)

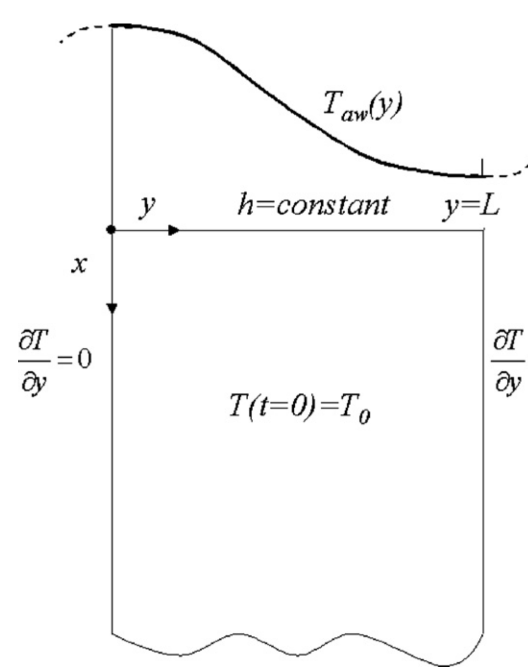

Fig. 1. Description of analysis model.

and the boundary conditions:

$$x = 0: \quad -k\frac{\partial T}{\partial x} = h(T_{\mathrm{aw}}(y) - T) \tag{3b}$$

$$x \to \infty: \quad T(x, y, t) \to T_0 \tag{3c}$$

The sidewalls will be assumed to be adiabatic assuming a periodic adiabatic wall temperature distribution as shown in Fig. 1, which is typical for film cooling investigations with rows of holes.

$$y = 0: \ \frac{\partial T}{\partial y} = 0 \quad \text{and} \quad y = L: \ \frac{\partial T}{\partial y} = 0 \tag{3d}$$

The adiabatic wall temperature distribution will be described by a cosine-series:

$$T_{\mathrm{aw}}(y) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\pi y/L) \tag{4}$$

Introducing the variables:

$$\theta = T - T_0, \quad \xi = x/L, \quad \eta = y/L, \quad \tau = at/L^2,$$
$$Bi = hL/k \tag{5}$$

the problem can be formulated for the region $0 \leqslant \xi < \infty$; $0 \leqslant \eta \leqslant 1$ by:

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \xi^2} + \frac{\partial^2 \theta}{\partial \eta^2} \tag{6}$$

$$\theta(\xi, \eta, \tau = 0) = 0 \tag{6a}$$

$$\xi = 0: -\frac{\partial \theta}{\partial \xi} = Bi(\theta_{\mathrm{aw}}(\eta) - \theta) \tag{6b}$$

$$\xi \to \infty: \theta(\xi, \eta, \tau) \to 0 \tag{6c}$$

$$\eta = 0: \ \frac{\partial \theta}{\partial \eta} = 0 \quad \text{and} \quad \eta = 1: \frac{\partial \theta}{\partial \eta} = 0 \tag{6d}$$

$$\theta_{\mathrm{aw}}(\eta) = c_0 + \sum_{n=1}^{\infty} c_n \cos(n\pi\eta) \tag{7}$$

$$\text{with } c_0 = \int_0^1 \theta_{\mathrm{aw}}(\eta)\mathrm{d}\eta \quad \text{and}$$
$$c_n = 2\int_0^1 \theta_{\mathrm{aw}}(\eta)\cos(n\pi\eta)\mathrm{d}\eta, \quad n = 1, 2, \ldots \tag{7a}$$

Using for the temperature distribution the ansatz:

$$\theta(\xi, \eta, \tau) = A_0(\xi, \tau) + \sum_{n=1}^{\infty} A_n(\xi, \tau)\cos(n\pi\eta) \tag{8}$$

the boundary conditions (6d) are automatically satisfied and the initial condition (6a) changes to

$$\tau = 0: A_n = 0 \quad n = 0, 1, 2, \ldots \tag{8a}$$

Inserting the ansatz (8) into the differential equation (6), one obtains for the coefficients A_n the differential equation

$$\frac{\partial A_n}{\partial \tau} = \frac{\partial^2 A_n}{\partial \xi^2} - (n\pi)^2 A_n \tag{9}$$

and for the boundary conditions:

$$\xi = 0: \quad \frac{\partial A_n}{\partial \xi} = -Bi(c_n - A_n) \tag{9a}$$

$$\xi \to \infty: A_n \to 0 \tag{9b}$$

Therewith the problem to be solved is related to a one-dimensional situation for a semi-infinite solid.

Applying Laplace transforms, where p is the Laplace variable and putting $N = n\pi$, Eqs. (9), (9a) and (9b) transform to:

$$p\bar{A}_n = \frac{\mathrm{d}^2\bar{A}_n}{\mathrm{d}\xi^2} - N^2\bar{A}_n,$$
$$\xi = 0: \quad \frac{\mathrm{d}\bar{A}_n}{\mathrm{d}\xi} = -Bi(\frac{c_n}{p} - \bar{A}_n),$$
$$\xi \to \infty: \quad \bar{A}_n \to 0 \tag{10}$$

where the over-bar denotes the variable in the Laplace domain.

The solution for this ordinary differential equation problem is given by:

$$\bar{A}_n(\xi, p) = \frac{c_n}{p}\frac{Bi}{\sqrt{p + N^2} + Bi}\exp\left(-\sqrt{p + N^2}\xi\right) \tag{11}$$

For the transient experiments considered here, the value of interest is the wall surface temperature at $\xi = 0$.

$$\bar{A}_n(0, p) = \frac{c_n}{p}Bi\frac{1}{\sqrt{p + N^2} + Bi}$$
$$= \frac{c_n}{p}Bi\frac{\sqrt{p + N^2}}{p + N^2 - Bi^2} - \frac{c_n}{p}Bi^2\frac{1}{p + N^2 - Bi^2} \tag{12}$$

Using partial fraction separation for the terms on the right hand side of Eq. (12), one obtains:

$$\bar{A}_n(0, p) = c_nBi\frac{\sqrt{p + N^2}}{p(N^2 - Bi^2)} - c_nBi\frac{\sqrt{p + N^2}}{(N^2 - Bi^2)(p + N^2 - Bi^2)}$$
$$- \frac{c_nBi^2}{p(N^2 - Bi^2)} + \frac{c_nBi^2}{(N^2 - Bi^2)(p + N^2 - Bi^2)} \tag{13}$$

which can be back-transformed using correspondence tables (e.g. [9]).

The solution in the time domain is then given by:

$$A_n(0, \tau) = \frac{c_nBi}{(N^2 - Bi^2)}N\mathrm{erf}\left(N\sqrt{\tau}\right) - \frac{c_nBi^2}{(N^2 - Bi^2)}$$
$$\times [1 - \exp\{(Bi^2 - N^2)\tau\}\mathrm{erfc}(Bi\sqrt{\tau})] \tag{14}$$

and the solution for the wall surface temperature is:

$$\theta_{\mathrm{W}}(\eta, \tau) = \theta(0, \eta, \tau)$$
$$= c_0\left[1 - \exp\{Bi^2\tau\}\mathrm{erfc}(Bi\sqrt{\tau})\right]$$
$$+ \sum_{n=1}^{\infty}\left\{\frac{c_nBi}{(n^2\pi^2 - Bi^2)}n\pi\mathrm{erf}\left(n\pi\sqrt{\tau}\right) - \frac{c_nBi^2}{(n^2\pi^2 - Bi^2)}\right.$$
$$\left.\times\left[1 - \exp\{(Bi^2 - n^2\pi^2)\tau\}\mathrm{erfc}(Bi\sqrt{\tau})\right]\right\}\cos(n\pi\eta) \tag{15}$$

and describes the surface temperature history at each lateral position taking the lateral conduction effect due to fluid temperature variations into account. The first term on the right hand side gives the known solution for the one-dimensional case with constant heat transfer coefficient and constant adiabatic wall temperature for the convective boundary condition.

For the singular case $Bi = N_0 = n_0\pi$ with a positive integer n_0 a limiting process can be performed leading to:

$$A_{n_0}(0,\tau) = \frac{c_{n_0}}{2}\left[\frac{2Bi}{\sqrt{\pi}}\sqrt{\tau}\exp(-Bi^2\tau) + \mathrm{erf}(Bi\sqrt{\tau}) - 2Bi^2\tau\,\mathrm{erfc}(Bi\sqrt{\tau})\right] \quad Bi \to N_0 \tag{16}$$

which has to be taken into account accordingly in Eq. (15).

For large times ($\tau \to \infty$) the wall temperature distribution is given by:

$$\theta_{\mathrm{W}}(\eta, \tau \to \infty) = c_0 + \sum_{n=1}^{\infty} c_n \frac{Bi}{n\pi + Bi}\cos(n\pi\eta) \tag{17}$$

Eq. (17) shows, that only for Bi-numbers much larger than $n\pi$, the wall temperature distribution approaches the adiabatic wall temperature distribution in the steady state.

3. Illustrative Example

As an example, a situation similar to the experimental conditions as described by Ling et al. [8] or Ai et al. [10] will be considered for a row of film cooling holes. A Perspex test plate ($a = 1 \times 10^{-7}\,\mathrm{m^2/s}$, $k = 0.18\,\mathrm{W/(mK)}$) has an initial temperature of $T_0 = 20\,°\mathrm{C}$ and contains film cooling holes, which are spaced 15 mm apart. At the start of the experiment the main stream is heated to $T_{\mathrm{G}} = 70\,°\mathrm{C}$ and coolant at a temperature of $T_{\mathrm{C}} = 20\,°\mathrm{C}$ is injected through the film cooling holes. The heat transfer coefficient at a certain stream-wise position is assumed to be $h = 100\,\mathrm{W/(m^2K)}$ and a periodic distribution of the adiabatic wall temperature develops from the mid-spacing between two successive holes ($y = 0$) to the centreline of one hole ($y = L = 7.5$ mm) at this stream-wise position. The adiabatic wall temperature distribution there is described by:

$$T_{\mathrm{aw}}(y) = (50 + 15\cos(\pi y/L))^{\circ}\mathrm{C}$$

This temperature distribution is shown in Fig. 2 together with the adiabatic film cooling effectiveness determined from:

$$\eta_{\mathrm{aw}}(y/L) = \frac{T_{\mathrm{G}} - T_{\mathrm{aw}}(y/L)}{T_{\mathrm{G}} - T_{\mathrm{C}}} \tag{18}$$

and the wall cooling effectiveness in the steady state using Eq. (17):

$$\varepsilon_{\mathrm{w}}(y/L) = \frac{T_{\mathrm{G}} - T_{\mathrm{w}}(y/L, t \to \infty)}{T_{\mathrm{G}} - T_{\mathrm{C}}} \tag{19}$$

This shows the homogenizing effect lateral conduction has on the steady state temperature distribution at the wall surface.

Using Eq. (15) with two terms (c_0, c_1), the wall temperature history at each position is calculated. For the data

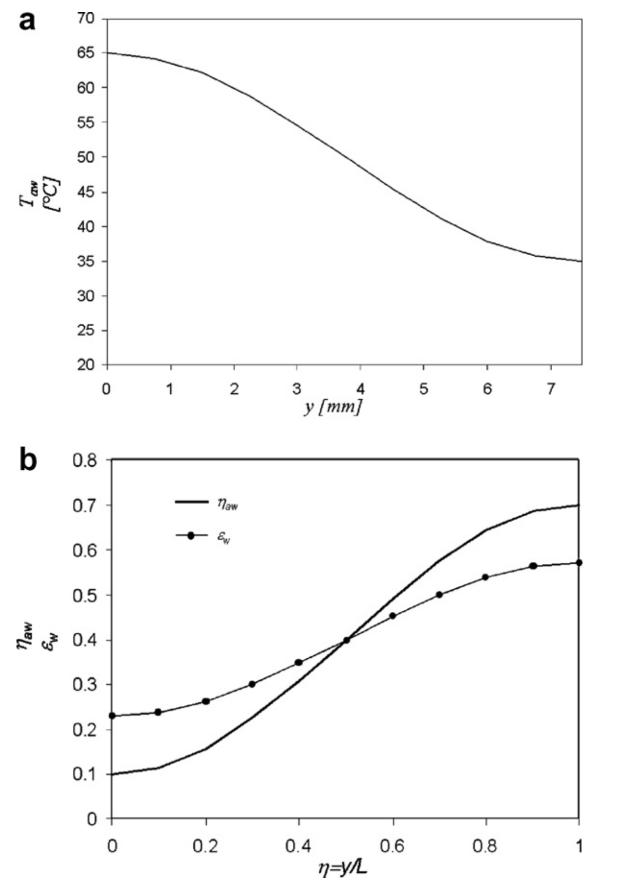

Fig. 2. (a) Prescribed distribution of adiabatic wall temperature (b) prescribed distribution of adiabatic film cooling effectiveness and wall effectiveness distribution for steady state.

evaluation it is assumed, that the surface is coated with two different narrow-band liquid crystals indicating at $T_{\mathrm{W1}} = 30\,°\mathrm{C}$ and $T_{\mathrm{W2}} = 33\,°\mathrm{C}$, respectively. From this the indication times for each position can be calculated, which are below 40 s for t_1 and below 100 s for t_2 for the two chosen liquid crystals.

First the data are evaluated using the one-dimensional solution (Eq. (1)) to solve for T_{aw} at each position and assuming that the heat transfer coefficient is known (possibly from another experiment with "isothermal conditions", e.g., $T_{\mathrm{G}} = T_{\mathrm{C}} > T_0$). The results of this data evaluation are shown in Fig. 3. For small values of the film cooling effectiveness (larger T_{aw}) the prescribed distribution is well matched. For these points the surface heating is relatively fast leading to small indication times, where lateral conduction has a minor influence. Depending on the chosen liquid crystal the differences for higher values of film cooling effectiveness (smaller T_{aw} and therefore later indication times) become larger.

Usually the heat transfer coefficient is not known and both values, T_{aw} and h, have to be determined simultaneously using e.g., Eq. (2). With this, the influence of the lateral conduction effects will influence both values, with the larger differences for the later indication times. This is

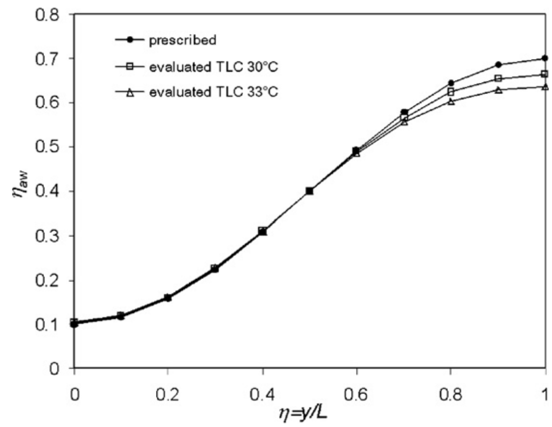

Fig. 3. Reevaluated data using one-dimensional model (Eq. (1)) and given heat transfer coefficient.

shown in Fig. 4. Due to the coupled evaluation for both parameters smaller values for the film cooling effectiveness are determined somewhat to large leading at the same time to over-predicted heat transfer coefficients. For the later indication times the values for both, η_{aw} and h, are seriously under-predicted due to the lateral conduction effects. The heat transfer coefficient in the regions of high adiabatic film cooling effectiveness is evaluated much too low. This corresponds to the findings of Ling et al. [8] where it is noted, that "the 1D model wrongly measures the effectiveness in region downstream of the film cooling hole" and showed, that the heat transfer levels in this region are significantly lower evaluated with the 1D model than with the 3D numerical procedure. Only at $y/L = 0.5$ the exact values are matched, since there is no net-effect of lateral conduction at this position due to the assumed periodic behavior of the adiabatic wall temperature $\left(\left.\frac{\mathrm{d}^2 T_{\mathrm{aw}}}{\mathrm{d}y^2}\right|_{y/L=0.5} = 0\right)$.

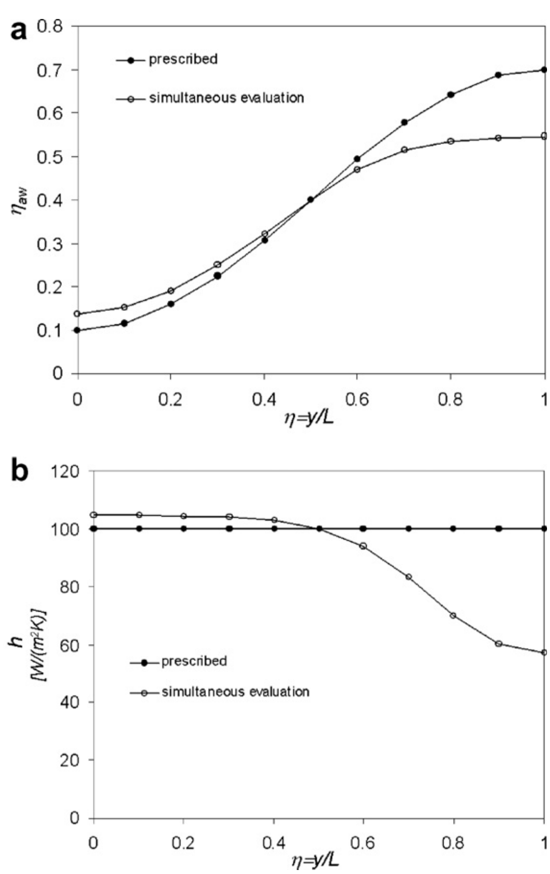

Fig. 4. Simultaneously evaluated data using Eq. (2).

4. Alternative data evaluation

Based on these investigations an alternative way for the data reduction can be given as follows. Instead of determining the parameters from the TLC indications at each position y_i $(i = 1, 2, \ldots, m)$ independently, a coupled evaluation should be used. In that case the overall error

$$\sum_{i=1}^{m} \left\{ \left[T_{\mathrm{w}}(C_n, t_{1i}, h_i) - T_{\mathrm{w1}}\right]^2 + \left[T_{\mathrm{w}}(C_n, t_{2i}, h_i) - T_{\mathrm{w2}}\right]^2 \right\} \quad (20)$$

should be minimized to determine the heat transfer coefficients h_i at each position and the coefficients C_n for the distribution of the adiabatic wall temperature using Eq. (15). In the given example (C_0, C_1) more indications than needed are available leading after the minimization of the overall error to the original prescribed values. A more general procedure could start with the evaluation of the data using Eq. (2). After this step, the obtained T_{aw} -distribution should be analyzed in form of a cosine-series, whereby areas for which adiabatic boundaries can be assumed will be identified and the number of coefficients C_n to be used will be determined. The final evaluation should now use all positions within the identified region in lateral direction and minimizing the overall error for all positions simultaneously leading to the local heat transfer coefficients and the respective description of the adiabatic wall temperature distribution over this region.

5. Conclusions

The presented analysis shows how lateral conduction effects due to fluid temperature variations might influence experimental data from film cooling investigations using transient measurement techniques and one-dimensional data evaluation processes. The importance of taking lateral conduction effects into account will depend on the actual experimental parameters. Although the analysis does not take into account possible variations in heat transfer coefficients, the given example for a particular situation illustrates that in film cooling experiments the effect of fluid temperature variations might be very significant. An alternative data evaluation procedure based on the given analysis is suggested, which might serve as an intermediate approach between the widely used one-dimensional assumption and processes using relatively time expensive three-dimensional unsteady numerical models.

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